Math 2471 – Fall 2017 Names Todd F.

Review for Test II

For full credit: use calculus to solve problems, circle answers, and **show all your work.**

1) Find all critical numbers of:

$f(x) = x^2(x+5)$ using calculus.

$f(x) = x^3 + 5x^2$

$f'(x) = 3x^2 + 10x$

Min/max @ $0 = (3x+10)x$

$x = -10/3$ or $x = 0$

2) Find the value of c to match the mean value on [-8, 8] of $f(x) = x^{-\frac{1}{3}}$.

Hint: mean value theorem!

This function is ~~NOT~~ continuous on $[-8, 8]$ so can't use MVT.

(or $C = \left(\sqrt[4]{\frac{-16}{3}}\right)^3$)

(NEED + check end pts.

3) Find the value of c to match the mean value on $[0, \pi]$ of $f(x) = \cos x$.

$f'(c) = \frac{f(b) - f(a)}{b - a}$

$= \frac{\cos\pi - \cos 0}{\pi - 0}$

$= \frac{-1-1}{\pi}$

$= -2/\pi$

$f'(x) = -\sin x$

$\frac{-2}{\pi} = -\sin c$

$\frac{2}{\pi} = \sin c$

$\sin^{-1}\left(\frac{2}{\pi}\right) = c$

$0.69 \approx c$

4) Find all maxima and minima on [0, 10] of $f(t) = t\sqrt{9-t}$ using calculus.

$f'(t) = (9-t)^{1/2} + -\frac{t}{2}(9-t)^{-1/2}$

min/max @ $0 = (9-t)^{1/2} - \frac{t}{2}(9-t)^{-1/2}$

(x by $\sqrt{9-t}$): $0 = 9 - t - t/2$

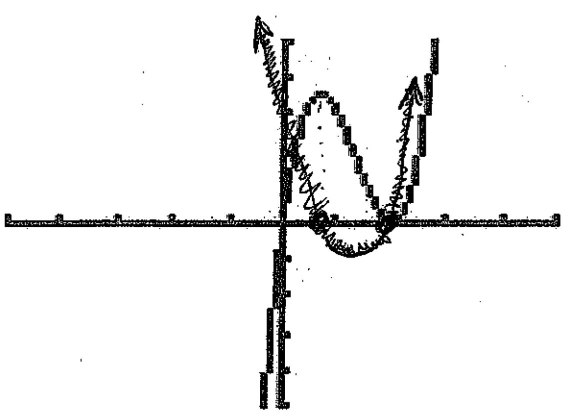
$3/2\, t = 9$

$t = 6$

Max at $t = 6$

Min @ $t = 0$

5) Below is a ~~picture of a function~~. Sketch the graph of this function's derivative on the same axes.



6) Find all intervals on which the function $f(x) = (x-2)^2(x-2)$ is increasing, decreasing, and all relative extrema.

$f'(x) = 0$ @ $x = 2$ (min/max)

$(x-2)(x-2)$

$f(x) = x^3 - 6x^2 + 12x - 8$

$f'(x) = 3x^2 - 12x + 12 = 3(x^2 - 4 + 4)$

$f''(x) = 6x - 12$

min $f''(2) < 0$

Increasing: $(-\infty, 2) \cup (2, \infty)$

Decreasing: $\emptyset$

Extrema:

Max: ∞ Min: $-\infty$

Inflection pt @ $f''(x) = 0 \Rightarrow 0 = 6x - 12$

$6x = 12$

$x = 2$

Intervals: $(-\infty, 2)$ & $(2, \infty)$

$f''(0) = -12 < 0$ ∴ concave down

$f''(3) = 6 > 0$ ∴ concave up

$f''(2) = 0$

∴ Not max or min

7) Find the points of inflection and discuss the concavity of $f(x) = x^3 - 6x^2 + 12x$.

$f'(x) = 3x^2 - 12x + 12$

$f''(x) = 6x - 12$

Inflection @ $f''(x) = 0$: $0 = 6x - 12$

$x = 2$

$(2, 8)$

$f''(1) = -6 < 0$ ← less than 2

$\therefore$ Concave down on $(-\infty, 2)$

$f''(3) = 6 > 0$

$\therefore$ Concave up on $(2, \infty)$

8) Find the limit as x approaches infinity for $f(x) = \dfrac{3x^2 - 8x}{x^2 + 2}$

$\Rightarrow \lim_{x \to \infty} \dfrac{(3x^2 - 8x)}{(x^2 + 2)} \dfrac{1/x^2}{1/x^2}$

$= \lim_{x \to \infty} \dfrac{3 - 8/x}{1 + 2/x^2}$

$= \dfrac{3 - 0}{1 + 0} = 3$

9) Find two positive numbers such that the second number is the reciprocal of the first and the sum is a minimum.

$x + 1/x = \text{sum}$

$f(x) = x + x^{-1}$

$f'(x) = 1 - x^{-2}$

min @ $f'(x) = 0$

$0 = 1 - x^{-2}$

$(x^{-2} = 1) \cdot x^2$

$\sqrt{1} = \sqrt{x^2}$ $1 \,\&\, 1$

$\pm 1 = x$

Want $\mathbb{Z}^+$ so $x = 1 \,\&\, 1/1$

10) Find the length and width of a rectangle with minimum perimeter and area of 100 sq. ft.

$2l + 2w = P$ $l \cdot w = 100 \Rightarrow l = 100w^{-1}$

$200w^{-1} + 2w = P$

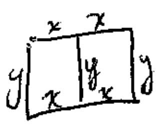
$-200w^{-2} + 2 = P'$ Min @ $P' = 0$

$-200w^{-2} + 2 = 0$

$2 = 200w^{-2}$

$w^2 = 100 \Rightarrow w = 10 \text{ ft} \;\&\; l = 10 \text{ ft}$

11) Suppose a rancher buys 1200 yards of fencing to enclose two adjacent rectangular corrals. What dimensions should be used so that the enclosed area will be a maximum?



$A = 2x \cdot y$

$P = 4x + 3y$

$1200 = 4x + 3y$

$\dfrac{1200 - 4x}{3} = y$

$A = 2x\left(\dfrac{1200 - 4x}{3}\right)$

$A = 800x - \frac{8}{3}x^2$

$dA/dx = 800 - \frac{16}{3}x$

$0 = 800 - \frac{16}{3}x$

$\frac{16}{3}x = 800$

$x = 150$ yds

$y = 200$ yds

12) Use Newton's method to find the zero in $f(x) = x^3 + x + 1$. The formula Newton found was: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$. Show each iteration.

$f'(x) = 3x^2 + 1$

Guess$_1$: $x = 2$ $2 - \frac{7}{13} = \frac{19}{13}$

Guess$_2$: $x = \frac{19}{13}$ $\frac{19}{13} - \frac{4.597633}{7.408840} \approx .840931449$

Guess$_3$: $x = .840931449 \leftarrow - \dfrac{2.54809715}{3.12149711} = .0246253439$

(Not very close because ~~there is~~ no zero).

13) Krusty the Klown is shot out of a circus cannon and his path can be modeled by the function $f(t) = 5t^3 - 20t^2 + 20t$ where t is the time in seconds and $f(t)$ is Krusty's height in meters for the first three seconds of his flight. Find Krusty's maximum and minimum heights.

$f'(t) = 15t^2 - 40t + 20$

Max/min @ $0 = 15t^2 - 40t + 20$

$0 = 5(3t - 2)(t - 2)$

Max @ $t = 2/3$ Min @ $t = 2$

$f''(t) = 30t - 40$

$0 = 30t - 40$

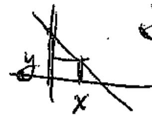
$4/3 = t$ Inflection

$(0, 4/3)$ concave down

$(4/3, 3)$ concave up.

14) Write a story for the three seconds of Krusty's Krazy Kannon show. Include discussions of extrema, concavity, and inflection points.

Krusty the Klown was launched out of a cannon @ ground level. He reached his max height at 2/3 of a second at ≈5.93 meters. Then he began falling back toward Earth. To demonstrate his showmanship skillz he turned on his turbo jet shoes at 4/3 of a second into the flight. He touched the ground at two seconds and the shoes carried him up, up, and away to a max height of 15 meters at 3 seconds. Then the shoes ran out of juice and Krusty Kame Krashing!



$y = -\frac{1}{2}x + 3$

19) A "Norman window" is a rectangle with a semicircle on top. Find the dimensions of a Norman window with perimeter of 16 ft and a maximum area.

$P = x + 2y + \frac{1}{2}(2\pi\frac{x}{2})$

$= x + 2y + \frac{1}{2}x\pi$

$16 = x + 2y + \frac{1}{2}x\pi$

$\frac{16 - x - \frac{1}{2}\pi x}{2} = y$

$A = x \cdot y + \frac{1}{2}\pi r^2$

$A = x(y) + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

$A = x\left(\frac{16}{2} - \frac{x}{2} - \frac{1}{4}\pi x\right) + \frac{1}{2}\pi\frac{x^2}{4}$

$A = 8x - \frac{1}{2}x^2 - \frac{1}{4}\pi x^2 + \frac{1}{8}\pi x^2$

$A = 8x - \frac{1}{2}x^2 - \frac{1}{8}\pi x^2$

$\frac{dA}{dx} = 8 - 1x - \frac{2}{8}\pi x$

Min/max @ $dA/dx = 0 \Rightarrow 0 = 8 - x - \frac{1}{4}\pi x$

$x + \frac{1}{4}\pi x = 8$

$x(1 + \frac{1}{4}\pi) = 8$

$x = \frac{8}{1 + \frac{1}{4}\pi} = \frac{32}{4+\pi}$ ft

$y = \frac{16}{4+\pi}$ ft

20) A rectangle (in the 1st quadrant) is bounded by the x-axis, y-axis, and line y=-0.5x + 3. Find the dimensions of the rectangle with maximum area.

$A = x \cdot y$

$A = x(-\frac{1}{2}x + 3)$

$A = -\frac{1}{2}x^2 + 3x$

$dA/dx = -1x + 3$

Max/min @ $0 = -1x + 3$

$1x = 3$

max @ $x = 3$

Dimensions: 3×1.5 units

21) Use $a(t)$= -9.8 m/sec./sec. due to gravity. Find an equation to represent the velocity and another equation to represent the height of an object using v_0 and h_0 for initial velocity and height.

$f''(x) = -9.8$

Velocity equation: $f'(x) = -9.8x + C$

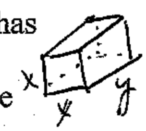
$f'(x) = -9.8x + V_0$

Height equation:

$f(x) = -4.9x^2 + V_0x + h_0$

(Working the derivative backward -- starting with $f''(x) = -9.8$ and finding the anti-derivative.)

22) A rectangular solid with a square base has a surface area of 337.5 square cm. Find the dimensions of the rectangle to maximize the volume.



$SA = 337.5\text{ cm}^2$

$SA = 2x^2 + 4xy$

$337.5 = 2x^2 + 4xy$

$\frac{337.5 - 2x^2}{4x} = y$

substitute

$V = x^2 \cdot y$

$V = x^2\left(\frac{337.5}{4x} - \frac{2x^2}{4x}\right)$

$V = 84.375x - \frac{1}{2}x^3$

$\frac{dV}{dx} = 84.375 - \frac{3}{2}x^2$

Min/max @ $\frac{dV}{dx} = 0 \Rightarrow 0 = 84.375 - \frac{3}{2}x^2$

$\frac{3}{2}x^2 = 84.375$

$x^2 = 56.25$

$x = 7.5$ cm

$y = \frac{337.5 - 2(7.5)^2}{4(7.5)}$

$= \frac{337.5 - 112.5}{30}$

$y = 7.5$ cm

$\boxed{\text{It's a } 7.5 \times 7.5 \times 7.5\text{ cm cube!}}$

15) Use the limit process to find the area under the curve of $y = x^2 + 2$ on [2, 5].

Sorry. this isn't coming until next week. :(

16) Given Newton's Method generates the next guess for a zero as: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, calculate three iterations for the zero of $y = 8x^5 + 3x - 2$ using the initial guess of 1.

	Guess	$-$	$\frac{f(x)}{f'(x)}$	$=$	Next guess
1)	1	$-$	$\frac{9}{43}$	$=$	$\frac{34}{43}$
2)	$\frac{34}{43}$	$-$	$\frac{2.844627148}{18.63514226}$	$\approx$	0.6380491404
3)	0.6380491404	$-$	$\frac{0.7601284548}{9.629434789}$	$\approx$	0.5591111267

17) Find all critical numbers and asymptotes of the function $f(x) = (x+1)^2(x-5)$

$f(x) = (x^2+2x+1)(x-5)$

$f(x) = x^3 + 2x^2 + x - 5x^2 - 10x - 5$

$f(x) = x^3 - 3x^2 - 9x - 5$

$f'(x) = 3x^2 - 6x - 9$

Critical values @ $f'(x) = 0$

$0 = 3x^2 - 6x - 9$

$0 = 3(x^2 - 2x - 3)$

$0 = 3(x-3)(x+1)$

$x = 3$ or $x = -1$

Critical numbers are $x = 3$ and $x = -1$, there are no asymptotes of $f(x)$.

18) Find all intervals on which the function $f(x) = (x+1)^2(x-5)$ is increasing, decreasing, and all relative extrema. Hey, I've seen you before! (#17)

$f''(x) = 6x - 6$

Inflection pts @ $f''(x) = 0$ → Intervals will be $(-\infty, 1)$ and $(1, \infty)$

$0 = 6x - 6$

$1 = x$

$0 \in$ $f''(0) = -6 < 0$ concave down on (Max @ -1)

$f''(2) = 6 > 0$ concave up on (Min @ 3)

Increasing: $(-\infty, -1) \cup (3, \infty)$

Decreasing: $(-1, 3)$

Extrema:

Max: $(-1, 0)$ Min: $(3, -32)$